

1. The value of C in $(0,2)$ satisfying the mean value theorem for the function $f(x) = x(x-1)^2$, $x \in [0,2]$ is equal to
- (a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) $\frac{1}{3}$ (d) $\frac{2}{3}$
2. $\frac{d}{dx} \left[\cos^2 \left(\cot^{-1} \sqrt{\frac{2+x}{2-x}} \right) \right]$ is
- (a) $-\frac{3}{4}$ (b) $-\frac{1}{2}$ (c) $\frac{1}{2}$ (d) $\frac{1}{4}$
3. For the function $f(x) = x^3 - 6x^2 + 12x - 3$; $x = 2$ is
- (a) a point of minimum (b) a point of inflexion
(c) not a critical point (d) a point of maximum
4. The function $f(x) = |\cos x|$ is
- (a) everywhere continuous and differentiable
(b) everywhere continuous but not differentiable at odd multiples of $\frac{\pi}{2}$
(c) neither continuous nor differentiable at $(2n+1)\frac{\pi}{2}$, $n \in \mathbb{Z}$
(d) not differentiable everywhere
5. If $y = 2x^{3x}$, then $\frac{dy}{dx}$ at $x = 1$ is
- (a) 2 (b) 6 (c) 3 (d) 1
6. Let the function satisfy the equation $f(x+y) = f(x)f(y)$ for all $x, y \in \mathbb{R}$, where $f(0) \neq 0$.
If $f(5) = 3$ and $f'(0) = 2$, then $f'(5)$ is
- (a) 6 (b) 0 (c) 5 (d) -6
7. $\int \frac{1}{x[6(\log x)^2 + 7\log x + 2]} dx =$
- (a) $\frac{1}{2} \log \left| \frac{2\log x + 1}{3\log x + 2} \right| + C$ (b) $\log \left| \frac{2\log x + 1}{3\log x + 2} \right| + C$
(c) $\log \left| \frac{3\log x + 2}{2\log x + 1} \right| + C$ (d) $\frac{1}{2} \log \left| \frac{3\log x + 2}{2\log x + 1} \right| + C$
8. $\int \frac{\sin \frac{5x}{2}}{\sin \frac{x}{2}} dx =$
- (a) $2x + \sin x + 2\sin 2x + C$ (b) $x + 2\sin x + 2\sin 2x + C$
(c) $x + 2\sin x + \sin 2x + C$ (d) $2x + \sin x + \sin 2x + C$
9. $\int_1^5 (|x-3| + |1-x|) dx =$

- (a) 12 (b) $\frac{5}{6}$ (c) 21 (d) 10

10. $\lim_{n \rightarrow \infty} \left(\frac{n}{n^2+1^2} + \frac{n}{n^2+2^2} + \frac{n}{n^2+3^2} + \dots + \frac{1}{5n} \right) =$

- (a) $\frac{\pi}{4}$ (b) $\tan^{-1}3$ (c) $\tan^{-1}2$ (d) $\frac{\pi}{2}$

11. The area of the region bounded by the line $y = 3x$ and the curve $y = x^2$ in sq. units is

- (a) 10 (b) $\frac{9}{2}$ (c) 9 (d) 5

12. The area of the region bounded by the line $y = x$ and the curve $y = x^3$ is

- (a) 0.2 sq. units (b) 0.3 sq. units
(c) 0.4 sq. units (d) 0.5 sq. units

13. If $\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar vectors and p, q, r are vectors defined by

$$\vec{p} = \frac{\vec{b} \times \vec{c}}{[\vec{a}\vec{b}\vec{c}]}, \vec{q} = \frac{\vec{c} \times \vec{a}}{[\vec{a}\vec{b}\vec{c}]}, \vec{r} = \frac{\vec{a} \times \vec{b}}{[\vec{a}\vec{b}\vec{c}]}, \text{ then } (\vec{a} + \vec{b}) \cdot \vec{p} + (\vec{b} + \vec{c}) \cdot \vec{q} + (\vec{c} + \vec{a}) \cdot \vec{r} \text{ is}$$

- (a) 0 (b) 1 (c) 2 (d) 3

14. If lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{y-5}{1} = \frac{z-6}{-5}$ are mutually perpendicular, then k is equal to

- (a) $-\frac{10}{7}$ (b) $-\frac{7}{10}$ (c) -10 (d) -7

15. The distance between the two planes $2x + 3y + 4z = 4$ and $4x + 6y + 8z = 12$ is

- (a) 2 units (b) 8 units (c) $\frac{2}{\sqrt{29}}$ units (d) 4 units

16. The sine of the angle between the straight line $\frac{x-2}{3} = \frac{y-3}{4} = \frac{4-z}{-5}$ and the plane $2x - 2y + z = 5$ is

- (a) $\frac{1}{5\sqrt{2}}$ (b) $\frac{2}{5\sqrt{2}}$ (c) $\frac{3}{50}$ (d) $\frac{3}{\sqrt{50}}$

17. The equation $xy = 0$ in three-dimensional space represents

- (a) a pair of straight lines (b) a plane
(c) a pair of planes at right angles (d) a pair of parallel planes

18. The plane containing the point $(3, 2, 0)$ and the line $\frac{x-3}{1} = \frac{y-6}{5} = \frac{z-4}{4}$ is

- (a) $x - y + z = 1$ (b) $x + y + z = 5$
(c) $x + 2y - z = 1$ (d) $2x - y + z = 5$

19. Corner points of the feasible region for an LPP are $(0, 2), (3, 0), (6, 0), (6, 8)$ and $(0, 5)$. Let $z = 4x + 6y$ be the objective function. The minimum value of z occurs at

- (a) Only $(0, 2)$ (b) Only $(3, 0)$
(c) The mid-point of the line segment joining the points $(0, 2)$ and $(3, 0)$

(d) Any point on the line segment joining the points (0,2) and (3,0)

20. A die is thrown 10 times. The probability that an odd number will come up at least once is

- (a) $\frac{11}{1024}$ (b) $\frac{1013}{1024}$ (c) $\frac{1023}{1024}$ (d) $\frac{1}{1024}$

21. A random variable X has the following probability distribution :

X	0	1	2
P(X)	$\frac{25}{36}$	k	$\frac{1}{36}$

If the mean of the random variable X is $\frac{1}{3}$, then the variance is

- (a) $\frac{1}{18}$ (b) $\frac{5}{18}$ (c) $\frac{7}{18}$ (d) $\frac{11}{18}$

22. If a random variable X follows the binomial distribution with parameters $n=5$, p and $P(X=2)=9P(X=3)$, then p is equal to

- (a) 10 (b) $\frac{1}{10}$ (c) 5 (d) $\frac{1}{5}$

23. Two finite sets have m and n elements respectively. The total number of subsets of the first set is 56 more than the total number of subsets of the second set. The values of m and n respectively are

- (a) 7, 6 (b) 5,1 (c) 6, 3 (d) 8,7

24. If $[x]^2 - 5[x] + 6 = 0$, where $[x]$ denotes the greatest integer function, then

- (a) $x \in [3,4]$ (b) $x \in [2,4)$ (c) $x \in [2,3]$ (d) $x \in (2,3]$

25. If in two circles, arcs of the same length subtend angles 30° and 78° at the centre, then the ratio of their radii is

- (a) $\frac{5}{13}$ (b) $\frac{13}{5}$ (c) $\frac{13}{4}$ (d) $\frac{4}{13}$

26. If ΔABC is right angled at C , then the value of $\tan A + \tan B$ is

- (a) $a + b$ (b) $\frac{a^2}{bc}$ (c) $\frac{c^2}{ab}$ (d) $\frac{b^2}{ac}$

27. The real value of ' α ' for which $\frac{1 - i \sin \alpha}{1 + 2i \sin \alpha}$ is purely real is

- (a) $(n+1)\frac{\pi}{2}, n \in \mathbb{N}$ (b) $(2n+1)\frac{\pi}{2}, n \in \mathbb{N}$
(c) $n\pi, n \in \mathbb{N}$ (d) $(2n-1)\frac{\pi}{2}, n \in \mathbb{N}$

28. The length of a rectangle is five times the breadth. If the minimum perimeter of the rectangle is 180 cm, then

- (a) Breadth ≤ 15 cm (b) Breadth ≥ 15 cm
(c) Length ≤ 15 cm (d) Length = 15 cm

29. The value of ${}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3 + {}^{46}C_3 + {}^{45}C_3 + {}^{45}C_4$ is

40. Let $(\text{gof})(x) = \sin x$ and $(f \circ g)(x) = (\sin \sqrt{x})^2$. Then

(a) $f(x) = \sin^2 x, g(x) = x$

(b) $f(x) = \sin \sqrt{x}, g(x) = \sqrt{x}$

(c) $f(x) = \sin^2 x, g(x) = \sqrt{x}$

(d) $f(x) = \sin \sqrt{x}, g(x) = x^2$

41. Let $A = \{2, 3, 4, 5, \dots, 16, 17, 18\}$. Let R be the relation on the set A of ordered pairs of positive integers defined by $(a, b) R (c, d)$ if and only if $ad = bc$ for all $(a, b), (c, d)$ in $A \times A$. Then the number of ordered pairs of the equivalence class of $(3, 2)$ is

(a) 4

(b) 5

(c) 6

(d) 7

42. If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = 3\pi$, then $x(y+z) + y(z+x) + z(x+y)$ equals to

(a) 0

(b) 1

(c) 6

(d) 12

43. If $2\sin^{-1} x - 3\cos^{-1} x = 4, x \in [-1, 1]$ then $2\sin^{-1} x + 3\cos^{-1} x$ is equal to

(a) $\frac{4-6\pi}{5}$

(b) $\frac{6\pi-4}{5}$

(c) $\frac{3\pi}{2}$

(d) 0

44. If A is a square matrix such that $A^2 = A$, then $(I + A)^3$ is equal to

(a) $7A - I$

(b) $7A$

(c) $7A + I$

(d) $I - 7A$

45. If $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$, then A^{10} is equal to

(a) $2^8 A$

(b) $2^9 A$

(c) $2^{10} A$

(d) $2^{11} A$

46. If $f(x) = \begin{vmatrix} x-3 & 2x^2-18 & 2x^3-81 \\ x-5 & 2x^2-50 & 4x^3-500 \\ 1 & 2 & 3 \end{vmatrix}$, then $f(1) \cdot f(3) + f(3) \cdot f(5) + f(5) \cdot f(1)$ is

(a) -1

(b) 0

(c) 1

(d) 2

47. Let $f(x) = \begin{vmatrix} \cos x & x & 1 \\ 2\sin x & x & 2x \\ \sin x & x & x \end{vmatrix}$. Then $\lim_{x \rightarrow 0} \frac{f(x)}{x^2} =$

(a) -1

(b) 0

(c) 3

(d) 2

48. Which one of the following observations is correct for the features of logarithm function to any base $b > 1$?

(a) The domain of the logarithm function is R , the set of real numbers.

(b) The range of the logarithm function is R^+ , the set of all positive real numbers.

(c) The point $(1, 0)$ is always on the graph of the logarithm function.

(d) The graph of the logarithm function is decreasing as we move from left to right.

49. If $P = \begin{bmatrix} 1 & \alpha & 3 \\ 1 & 3 & 3 \\ 2 & 4 & 4 \end{bmatrix}$ is the adjoint of a 3×3 matrix A and $|A| = 4$, then α is equal to

(a) 4

(b) 5

(c) 11

(d) 0

50. If $A = \begin{vmatrix} x & 1 \\ 1 & x \end{vmatrix}$ and $B = \begin{vmatrix} x & 1 & 1 \\ 1 & x & 1 \\ 1 & 1 & x \end{vmatrix}$, then $\frac{dB}{dx}$ is
- (a) 3 A (b) -3 B (c) 3 B+1 (d) 1-3A
51. If $f(x) = xe^{x(1-x)}$ then $f(x)$ is
- (a) increasing in \mathbf{R} (b) decreasing in \mathbf{R}
(c) decreasing in $\left[-\frac{1}{2}, 1\right]$ (d) increasing in $\left[-\frac{1}{2}, 1\right]$
52. $\int \frac{\sin x}{3+4\cos^2 x} \cdot dx =$
- (a) $-\frac{1}{2\sqrt{3}} \tan^{-1}\left(\frac{2\cos x}{\sqrt{3}}\right) + C$ (b) $\frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{\cos x}{3}\right) + C$
(c) $\frac{1}{2\sqrt{3}} \tan^{-1}\left(\frac{\cos x}{3}\right) + C$ (d) $-\frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{2\cos x}{3}\right) + C$
53. $\int_{-\pi}^{\pi} (1-x^2) \sin x \cdot \cos^2 x dx =$
- (a) $\pi - \frac{\pi^2}{3}$ (b) $2\pi - \pi^3$ (c) $\pi - \frac{\pi^3}{2}$ (d) 0
54. The function $x^x; x > 0$ is strictly increasing at
- (a) $\forall x \in \mathbf{R}$ (b) $x < \frac{1}{e}$ (c) $x > \frac{1}{e}$ (d) $x < 0$
55. The maximum volume of the right circular cone with slant height 6 units is
- (a) $4\sqrt{3}\pi$ cubic units (b) $16\sqrt{3}\pi$ cubic units
(c) $3\sqrt{3}\pi$ cubic units (d) $6\sqrt{3}\pi$ cubic units
56. The vectors $\overline{AB} = 3i + 4k$ and $\overline{AC} = 5i - 2j + 4k$ are the sides of ΔABC . The length of the median through A is
- (a) $\sqrt{18}$ (b) $\sqrt{72}$ (c) $\sqrt{33}$ (d) $\sqrt{288}$
57. The volume of the parallelepiped whose co-terminous edges are $j+k, i+k$ and $i+j$ is
- (a) 6 cu.units (b) 2 cu.units (c) 4 cu.units (d) 3 cu.units
58. Let \vec{a} and \vec{b} be two unit vectors and θ is the angle between them. Then $\vec{a} + \vec{b}$ is a unit vector if
- (a) $\theta = \frac{\pi}{4}$ (b) $\theta = \frac{\pi}{3}$ (c) $\theta = \frac{2\pi}{3}$ (d) $\theta = \frac{\pi}{2}$
59. The solution of $e^{\frac{dy}{dx}} = x+1, y(0) = 3$ is
- (a) $y - 2 = x \log x - x$ (b) $y - x - 3 = x \log x$
(c) $y - x - 3 = (x+1) \log(x+1)$ (d) $y + x - 3 = (x+1) \log(x+1)$

60. The family of curves whose x and y intercepts of a tangent at any point are respectively double the x and y coordinates of that point is

(a) $xy = C$

(b) $x^2 + y^2 = C$

(c) $x^2 - y^2 = C$

(d) $\frac{y}{x} = C$